Self-consistency of the Dressed Electromagnetic Nucleon Current

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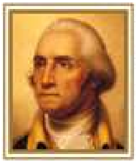


Language

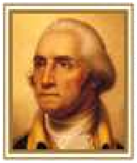
Whenever I say "photoprocess," **the photon may be real or virtual**. The formalism to be presented here applies to either case.

How to Read the Diagrams

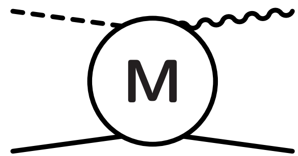
Time runs from right to left in all diagrams, i.e., the same direction as in matrix elements:

$$\langle \mathrm{final} | (\text{some operator}) | \mathrm{initial} \rangle$$

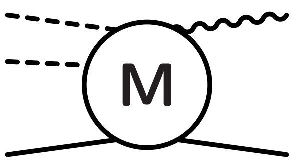
$$\Leftarrow \quad \textit{time} \quad \Leftarrow$$



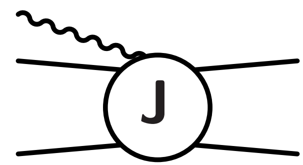
Introduction



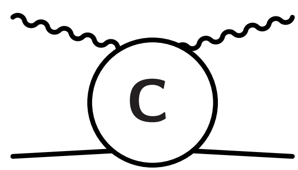
$\gamma N \to \pi N$



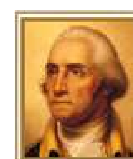
$\gamma N \to \pi\pi N$



$NN \to NN\gamma$



$\gamma N \to \gamma N$



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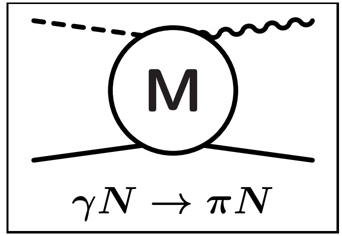
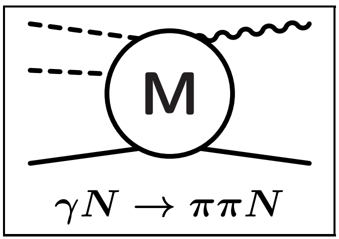
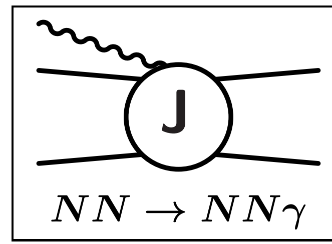
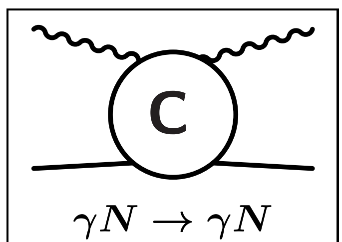
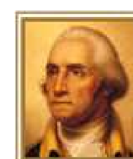




What is the common feature of these photo reactions?







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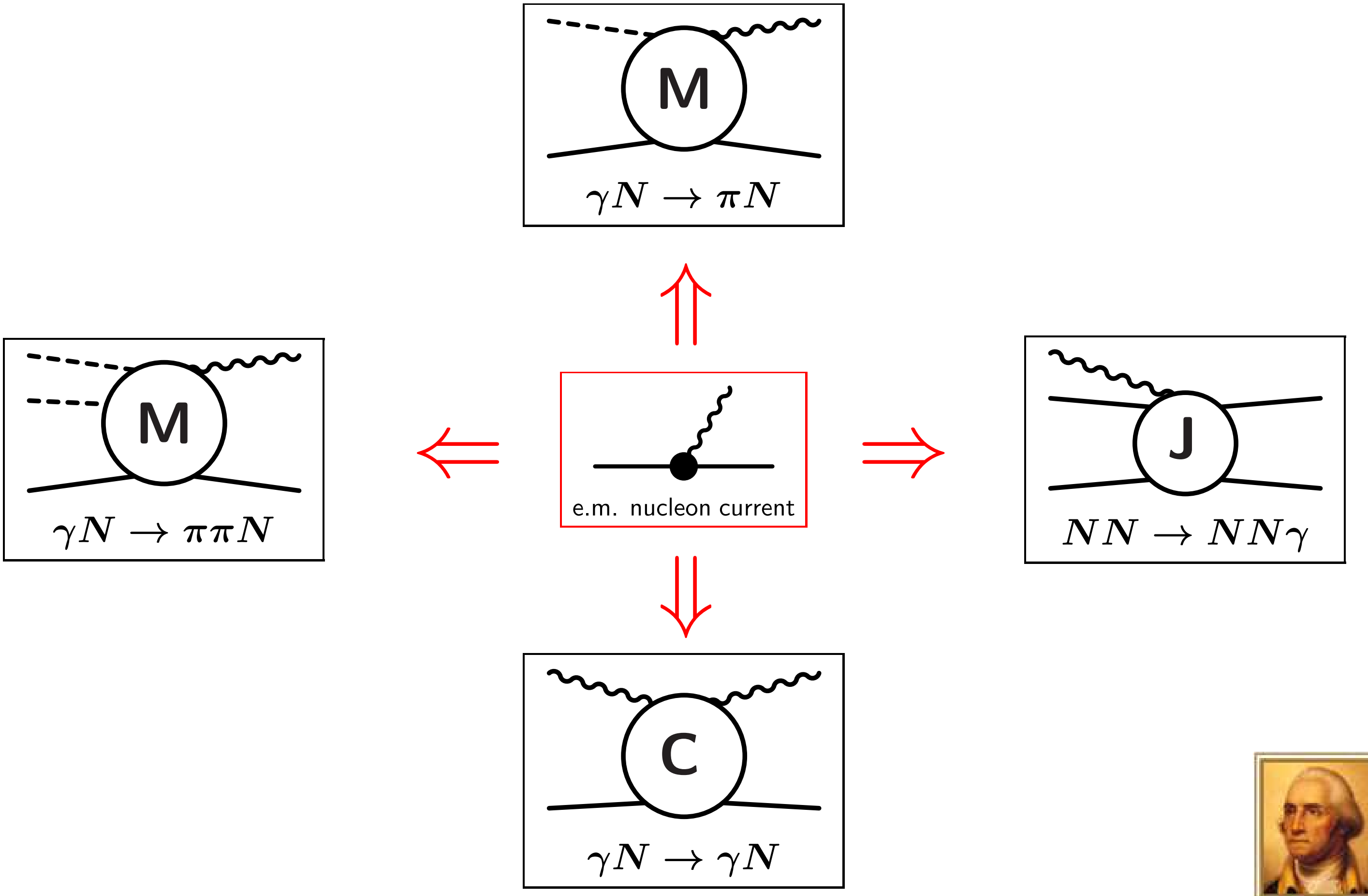
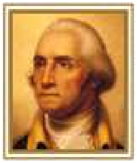


Electromagnetic Current J^μ of the Nucleon

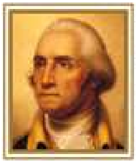
How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?



Electromagnetic Current J^μ of the Nucleon

How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?

- The most general Lorentz-covariant structure of J^μ requires **12 form factors**.
- Applying gauge invariance, this reduces to **8 form factors**.
- Applying time-reversal invariance, this reduces further to **6 form factors**.



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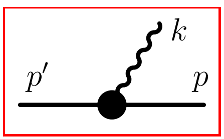
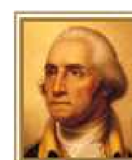
$$\gamma_{\mathrm{T}}^\mu = \gamma^\mu - k^\mu \frac{\not{k}}{k^2}$$

$$J^\mu(p',p) = e\Bigg[\delta_N\gamma^\mu + \delta_N\gamma_{\mathrm{T}}^\mu(F_1-1) + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N F_2 + \frac{S^{-1}(p')}{2m}\left(\gamma_{\mathrm{T}}^\mu f_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N f_2\right) + \left(\gamma_{\mathrm{T}}^\mu f_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N f_2\right)\frac{S^{-1}(p)}{2m} + \frac{S^{-1}(p')}{2m}\left(\gamma_{\mathrm{T}}^\mu g_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N g_2\right)\frac{S^{-1}(p)}{2m}\Bigg]$$



(Approximation)



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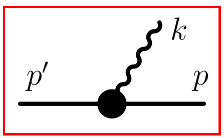
$$+\frac{S^{-1}(p')}{2m}\left(\gamma_{\rm T}^\mu f_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N f_2\right) + \left(\gamma_{\rm T}^\mu f_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N f_2\right)\frac{S^{-1}(p)}{2m}$$
$$\left.+\frac{S^{-1}(p')}{2m}\left(\gamma_{\rm T}^\mu g_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N g_2\right)\frac{S^{-1}(p)}{2m}\right]$$

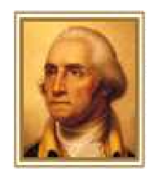


(Approximation)

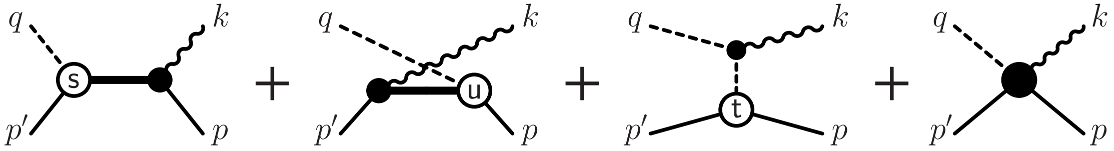
Constraints:

no kinematic singularity: $f_1(k^2) \xrightarrow{k^2=0} 0$ and $g_1(k^2) \xrightarrow{k^2=0} 0$

chiral-symmetry limit : $f_1 \to \dfrac{g_A - G_A(k^2)}{g_A}$ and $f_2 \to 1$



Implications of off-shell structure: Pion photoproduction



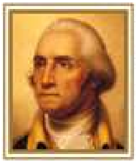
s-channel:

$$F_s S(p+k) J_i^\mu(p+k,p) = F_s S(p+k) \left(e\delta_i \gamma^\mu + \frac{i\sigma^{\mu\nu}k_\nu}{2m} e\kappa_i \right) + F_s \frac{i\sigma^{\mu\nu}k_\nu}{2m}\frac{e\kappa_i}{2m} f_{2i}$$

contact terms

u-channel:

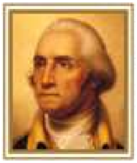
$$J_f^\mu(p',p'-k) S(p'-k) F_u = \left(e\delta_f \gamma^\mu + \frac{i\sigma^{\mu\nu}k_\nu}{2m} e\kappa_f \right) S(p'-k) F_u + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\frac{e\kappa_f}{2m} f_{2f} F_u$$



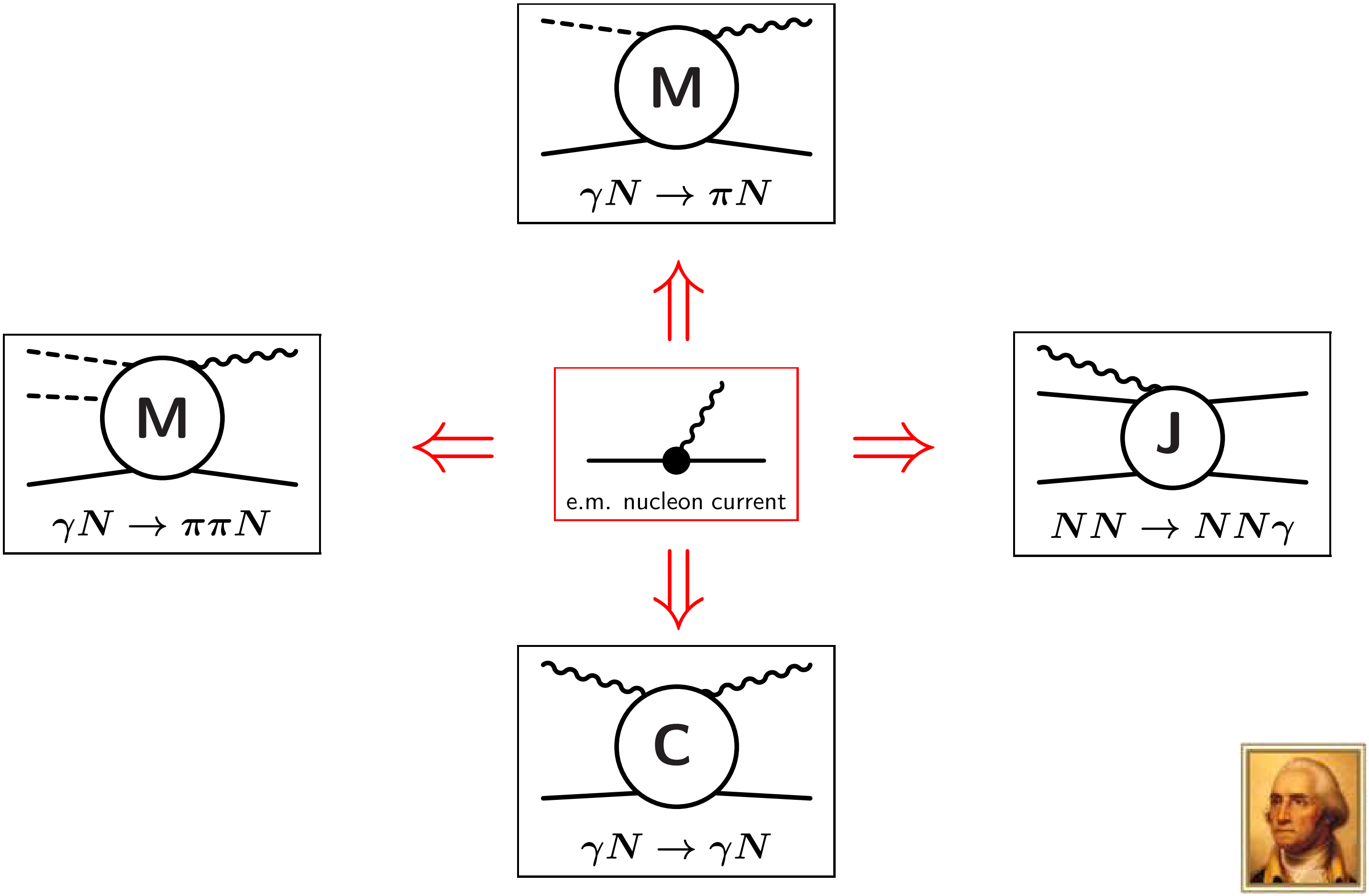
Electromagnetic Current J^μ of the Nucleon

- Photoprocesses, in general, require a more detailed description of J^μ.

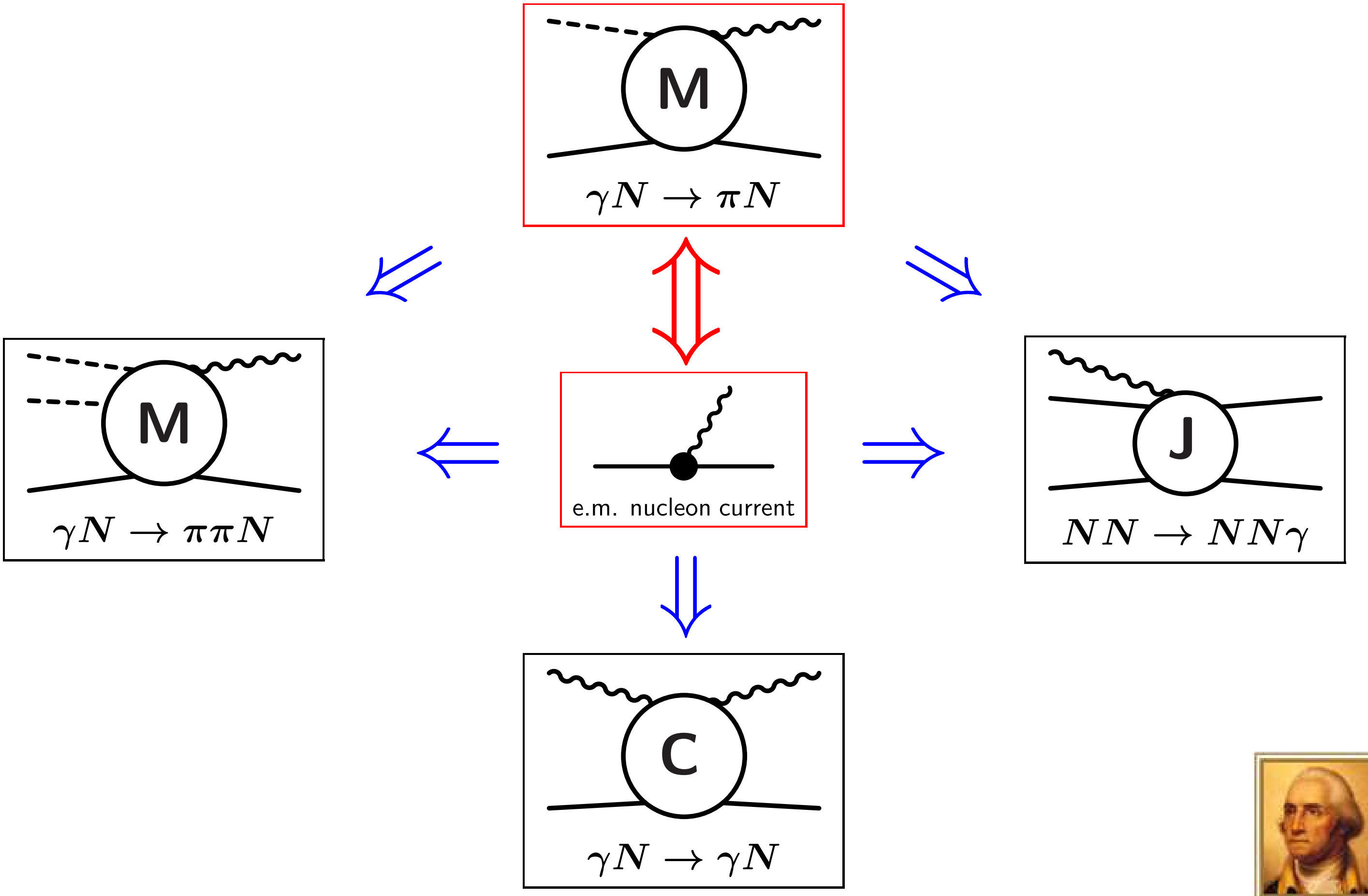
- The dynamical structures of the current J^μ can be determined by requiring self-consistency.



Introduction

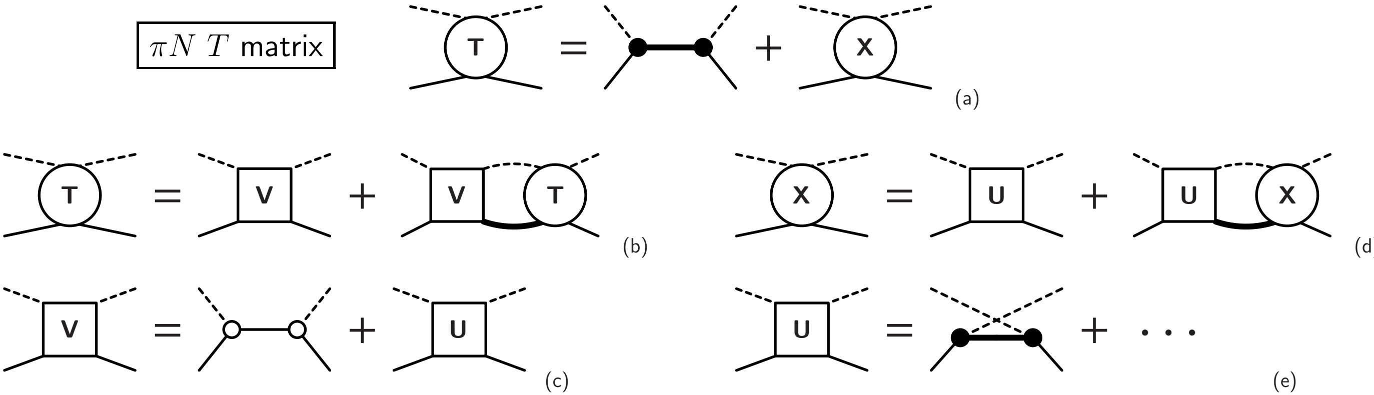


Dynamical Links between Photoprocesses



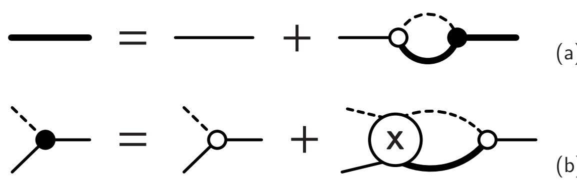
Pions, Nucleons, and Photons

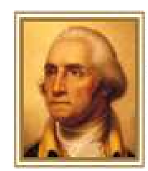
HH, PRC **56**, 2041 (1997)



dressed nucleon propagator

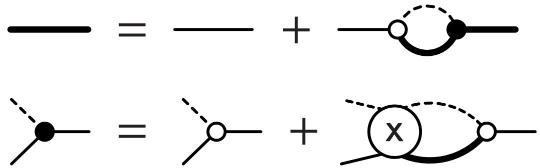
dressed πNN vertex





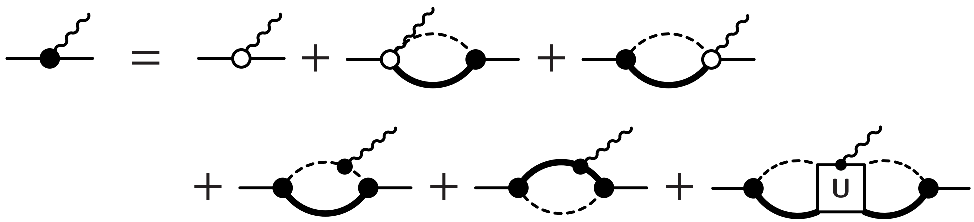
- Tower of *nonlinear* Dyson-Schwinger-type equations

Nucleon Current J^μ

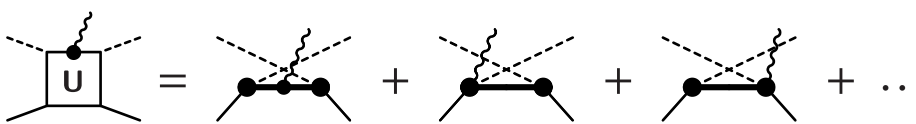


(a) (b)

- **Couple photon to dressed propagator:**

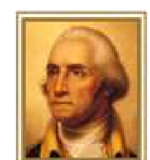


(a)



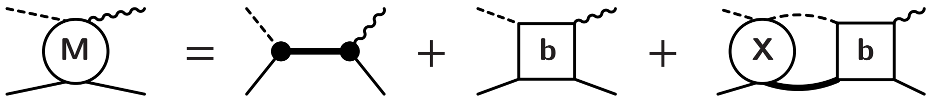
(b)

- Tower of *nonlinear* Dyson-Schwinger-type equations

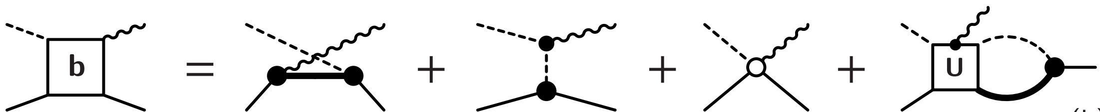


Pion Photoproduction

- **Pion-production current M^μ:**

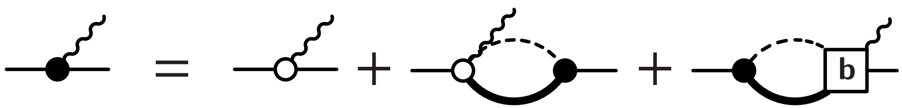


(a)

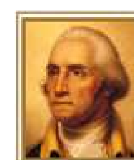


(b)

- **Nucleon current J^μ:**



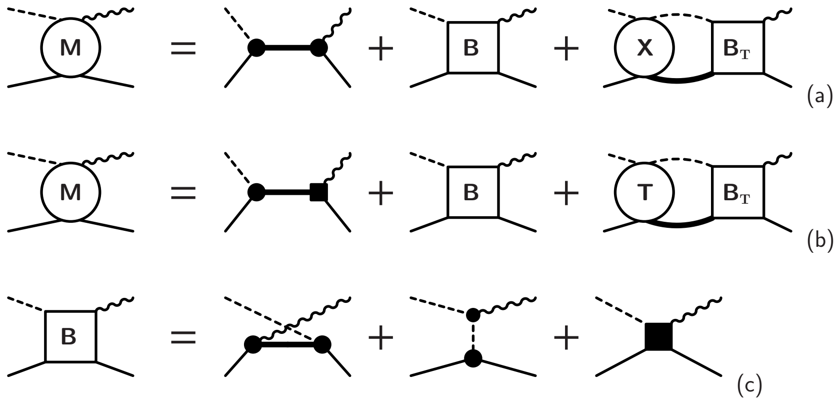
$\Rightarrow$ The internal structures of the dressed nucleon current can be understood by the dynamics of the pion production current.



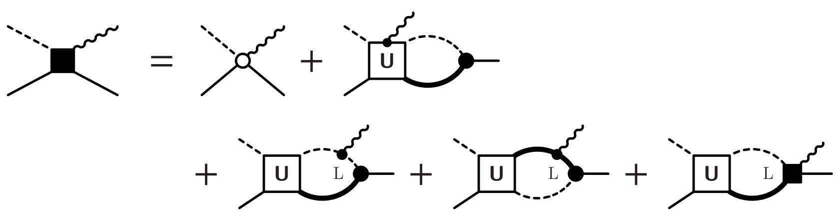
- Tower of *nonlinear* Dyson-Schwinger-type equations

Rewriting the Production Current

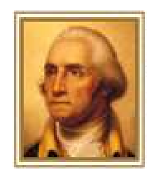
- **Pion-production current M^μ:**



- **Contact-type current M_c^μ:**

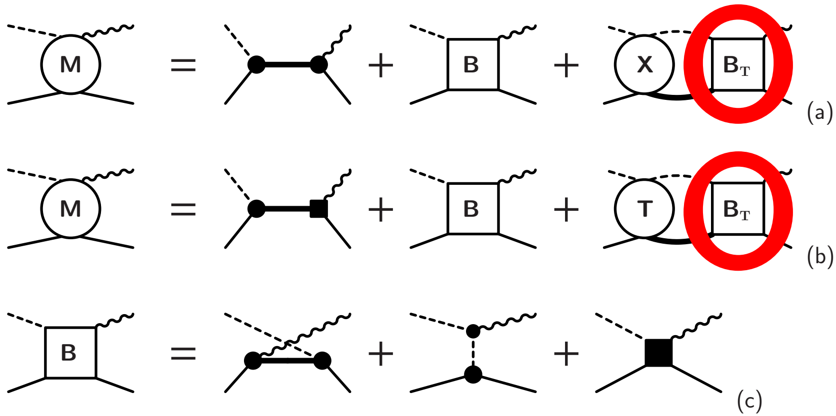


- Tower of *nonlinear* Dyson-Schwinger-type equations



Rewriting the Production Current

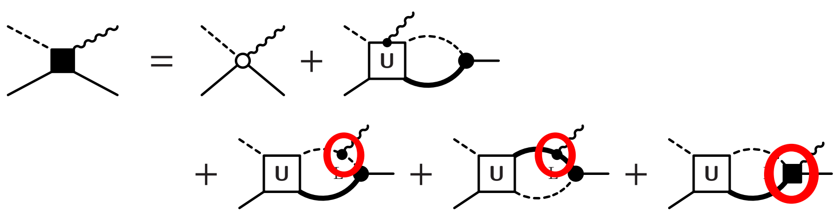
- **Pion-production current M^μ:**



transverse

(irrelevant for gauge invariance)

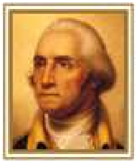
- **Contact-type current M_c^μ:**



partial integral equation

longitudinal

- Tower of *nonlinear* Dyson-Schwinger-type equations



Rewriting the Production Current

HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)

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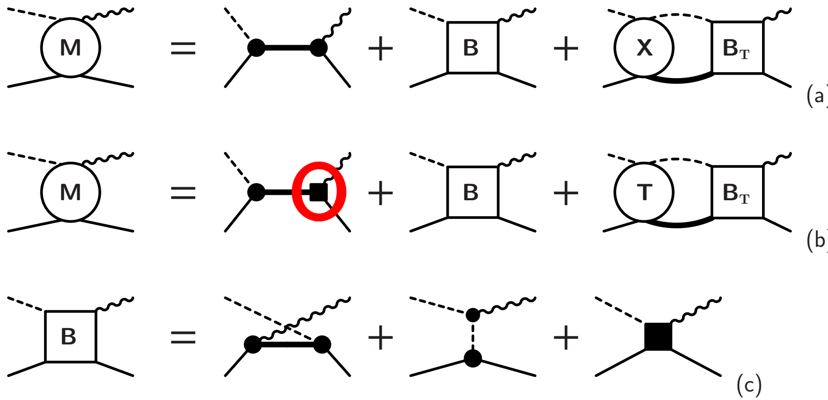

J_s^μ

not the full nucleon current

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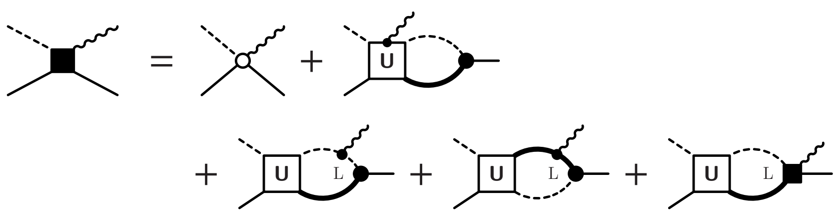
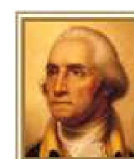
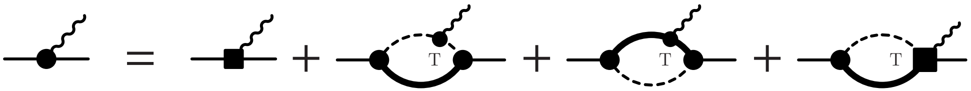


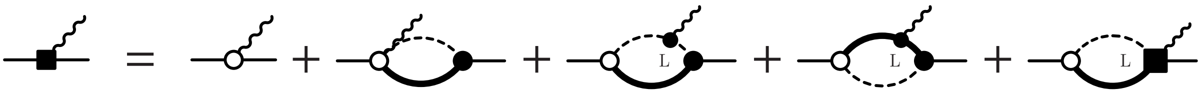
- Tower of *nonlinear* Dyson-Schwinger-type equations



Nucleon Current J^μ

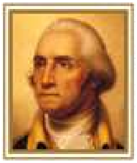


(a)

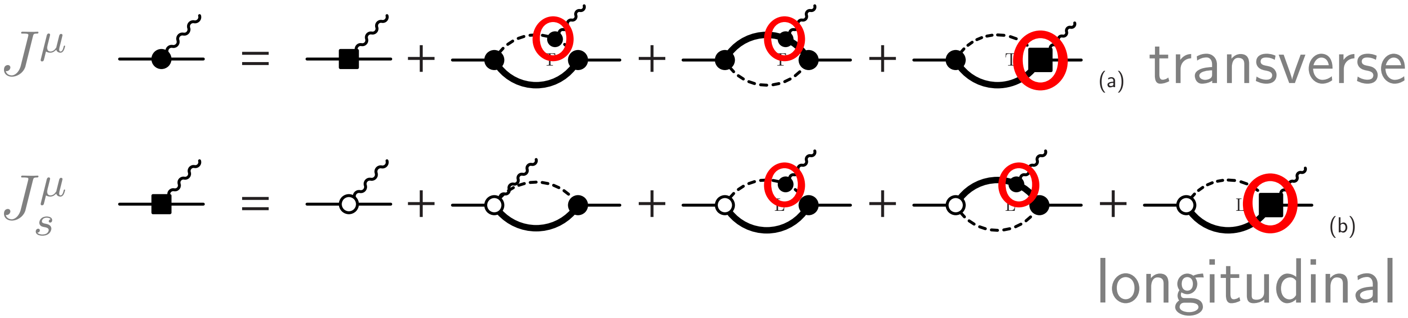


(b)

- Tower of *nonlinear* Dyson-Schwinger-type equations



Nucleon Current J^μ



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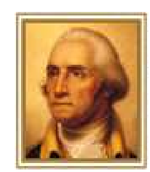

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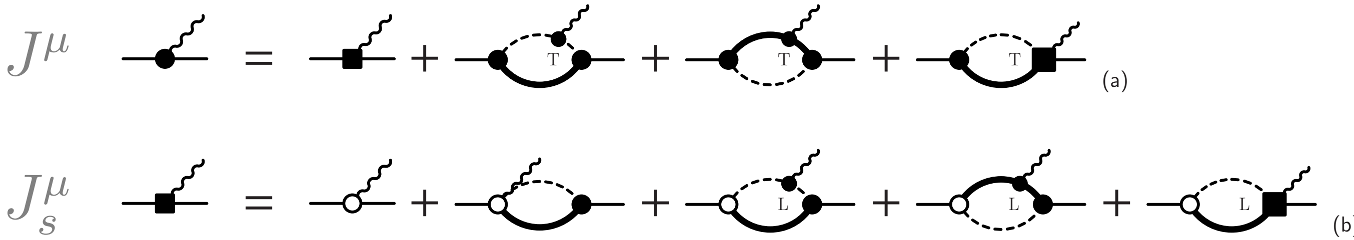
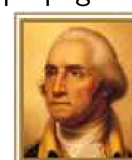
HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)



Gauge Invariance: Ward-Takahashi Identity (WTI)

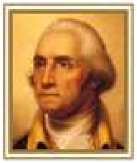
$$k_\mu J^\mu(p',p) = k_\mu J_s^\mu(p',p) = S^{-1}(p')Q_N - Q_N S^{-1}(p)$$

S: dressed nucleon propagator



Problems?

- Everything is exact!
- Everything is nonlinear!
- Everything is hideously complicated!



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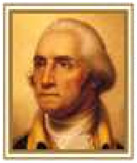
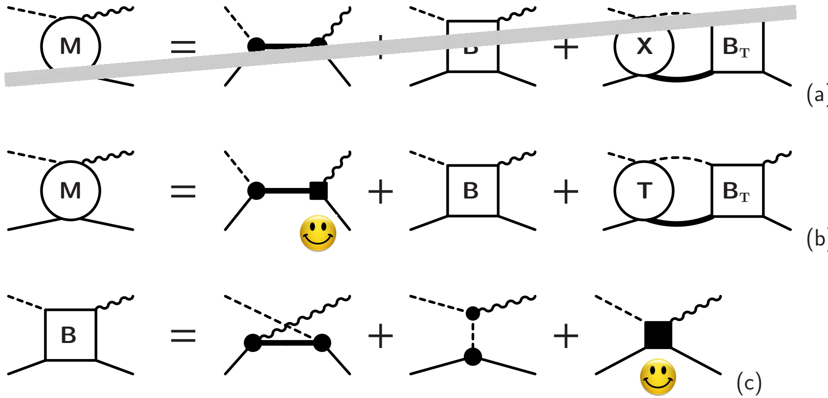
- Everything is nonlinear!
- Everything is hideously complicated!

But...

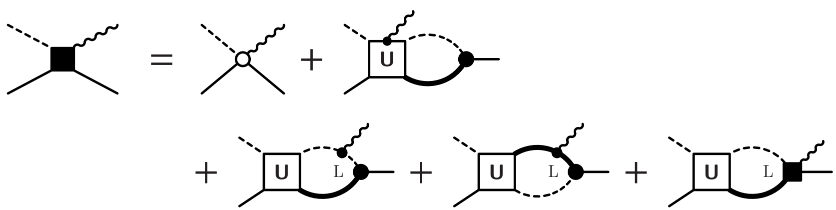


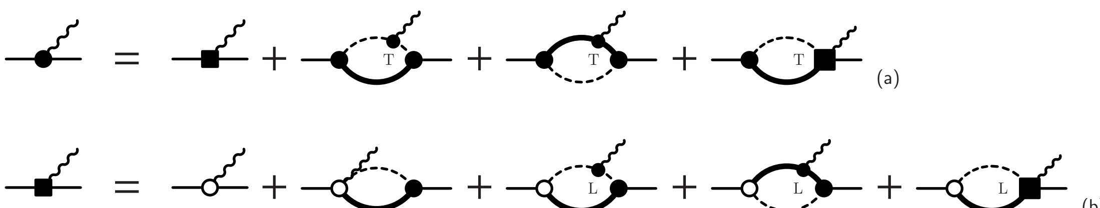
Let’s cut the Gordian knot!

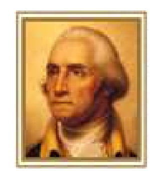
HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)



Do not use X.
Work with full T.

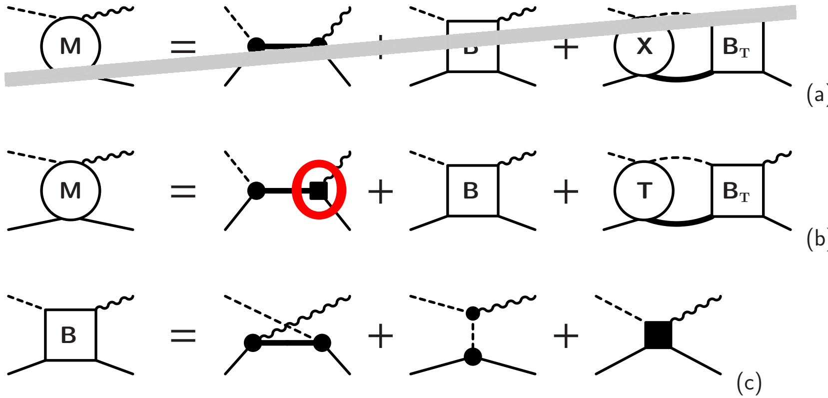






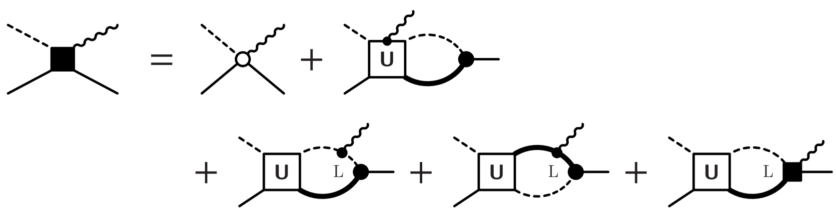
Cutting the Gordian Knot

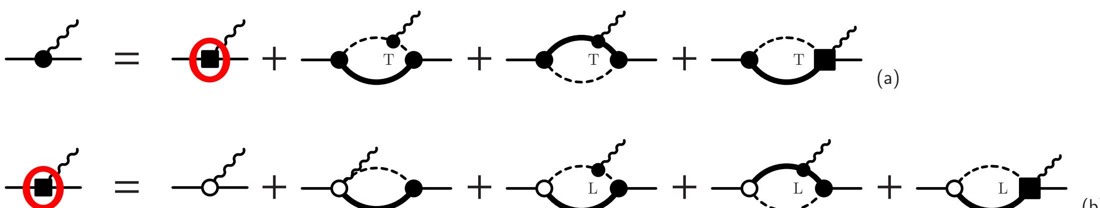
HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)



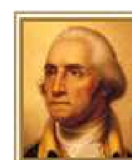
J^μ_s

not the full nucleon current



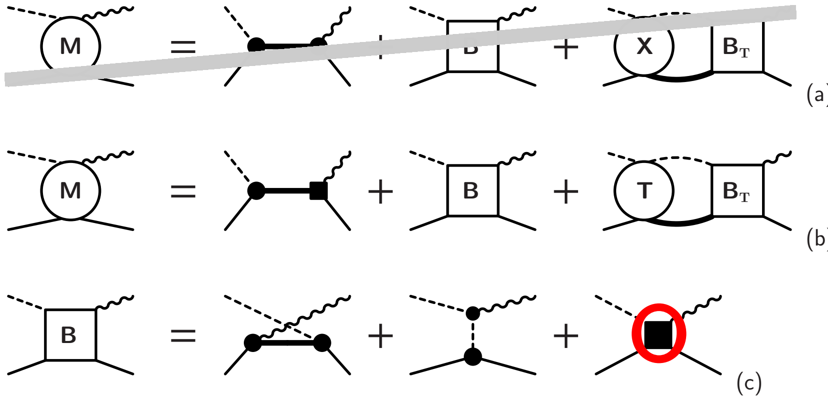


determine approximation by WTI for the nucleon current J^μ

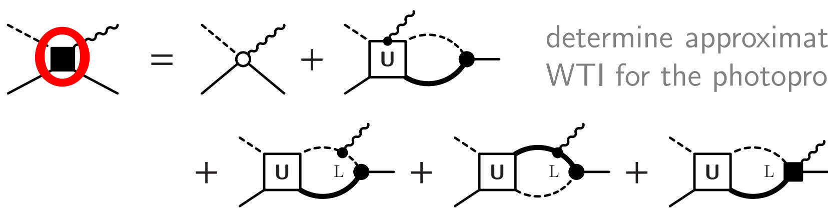


Cutting the Gordian Knot

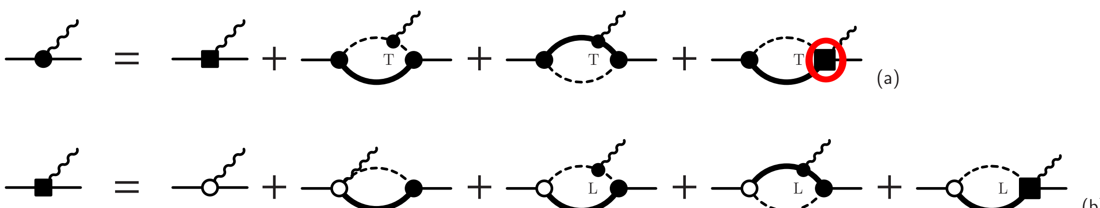
HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)

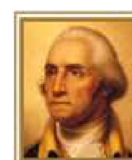


M_c^μ



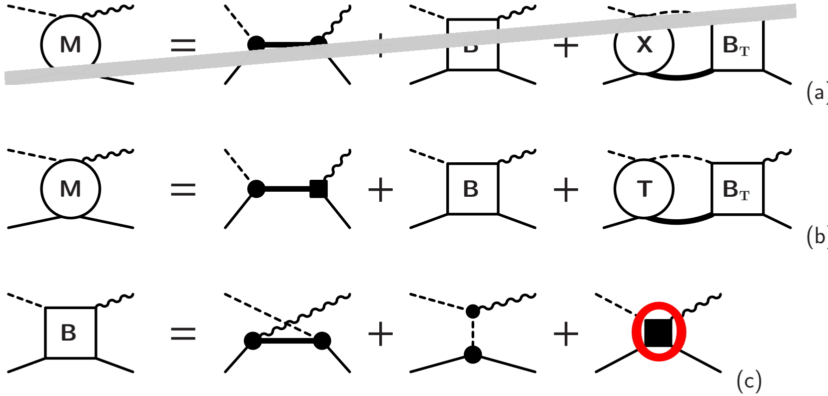
determine approximation of M_c^μ by generalized WTI for the photoproduction current M^μ





Approximating M_c^μ

HH, F. Huang, K. Nakayama, arXiv:1103.2065 [nucl-th] (2011)



$$M_c^\mu$$

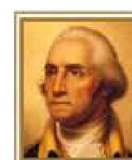
- **Lowest-order approximation in terms of phenomenological form factors:**

$$M_c^\mu = ge\gamma_5 \frac{i\sigma^{\mu\nu}k_\nu}{4m^2}\tilde{\kappa}_N - (1-\lambda)g\frac{\gamma_5\gamma^\mu}{2m}\tilde{F}_t e_\pi - G_\lambda \left[e_i \frac{(2p+k)^\mu}{s-p^2}\left(\tilde{F}_s - \hat{F}\right) \right.$$

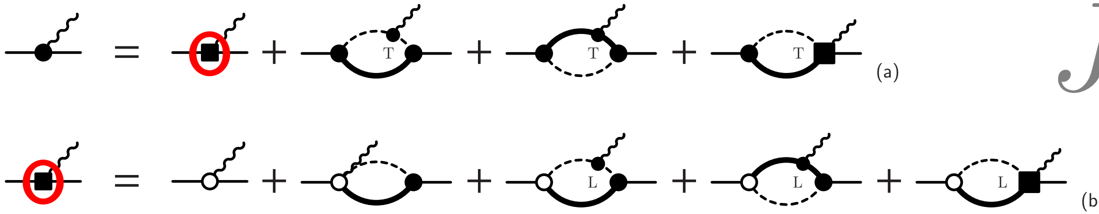
$$+ e_f \frac{(2p'-k)^\mu}{u-p'^2}\left(\tilde{F}_u - \hat{F}\right)$$

$$\left. + e_\pi \frac{(2q-k)^\mu}{t-q^2}\left(\tilde{F}_t - \hat{F}\right) \right]$$

Don't try to read the details. What is important is that this is a simple expression, easy to evaluate, and that it helps preserve gauge invariance of the entire production current.



Approximating J_s^μ



J_s^μ

determine approximation by WTI for the nucleon current J^μ

- **Approximate J_s^μ by the minimal current that reproduces the WTI:**

$S^{-1}(p) = \not{p}A(p^2) - mB(p^2)$

$$J_s^\mu(p',p) = (p'+p)^\mu \frac{S^{-1}(p')Q_N - Q_N S^{-1}(p)}{p'^2 - p^2} + \left[\gamma^\mu - \frac{(p'+p)^\mu}{p'^2-p^2}\not{k}\right] Q_N \frac{A(p'^2)+A(p^2)}{2}$$

Ball–Chiu:
Satisfies WTI
Nonsingular
Minimal
Unique!

- **Half on-shell:**

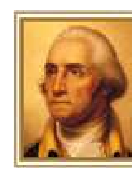
$$S J_s^\mu u = \left(\frac{1}{\not{p}+\not{k}-m} j_1^\mu + \frac{2m}{s-m^2} j_2^\mu\right) Q_N\, u(p)\ , \qquad \text{with} \qquad s = (p+k)^2$$

Exact!

- Auxiliary currents:

$$j_1^\mu = \gamma^\mu\ (1-\kappa_1) + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_1 \qquad\qquad j_2^\mu = \frac{(2p+k)^\mu}{2m}\kappa_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_2$$

Two parameters!



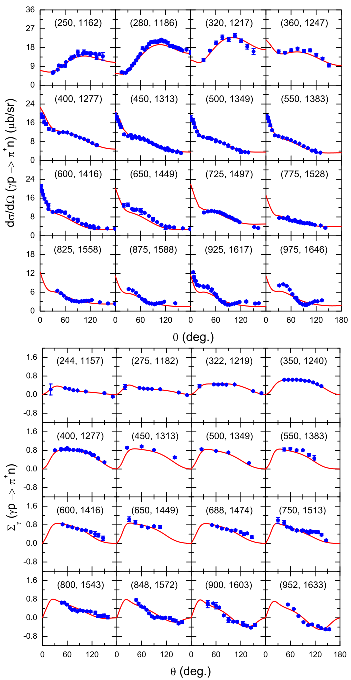
Does it work? — Yes!

- **Preliminary results for $\gamma N \to \pi N$**

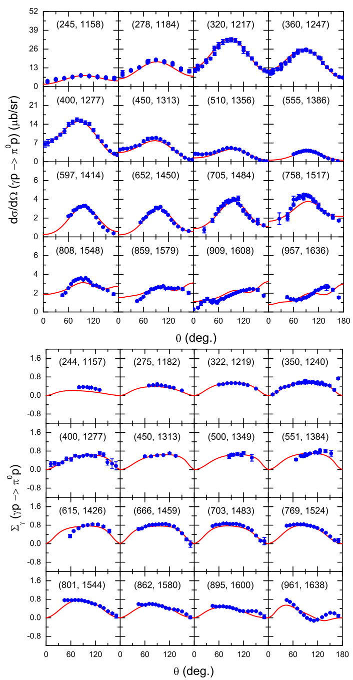
Fei Huang, Wednesday afternoon

$\frac{d\sigma}{d\Omega}$

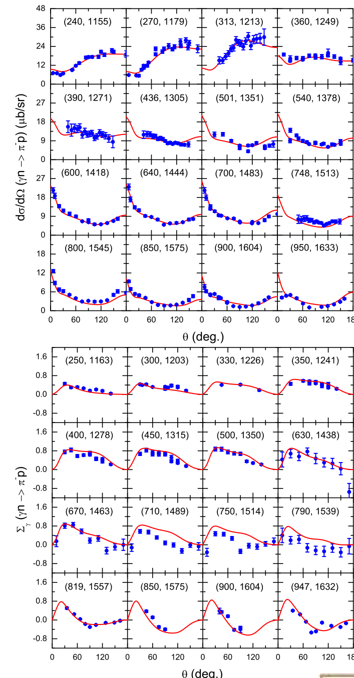
Σ



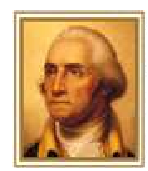
$\gamma p \to \pi^+ n$



$\gamma p \to \pi^0 p$



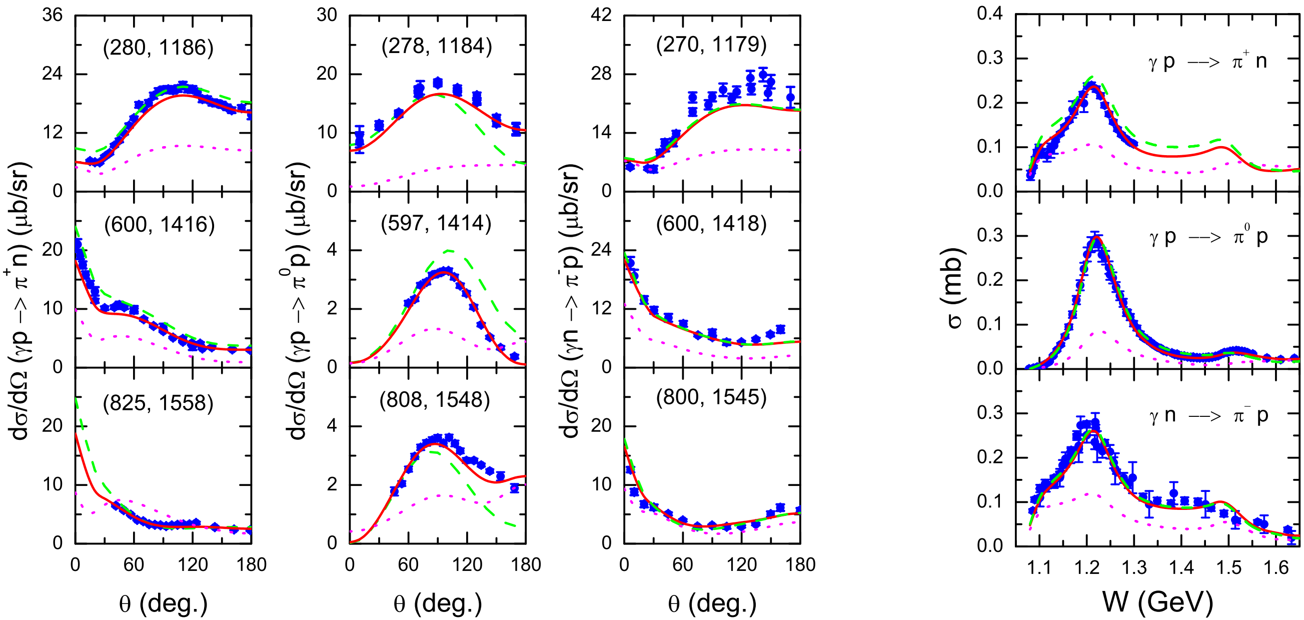
$\gamma n \to \pi^- p$



F. Huang, M. Döring, H. Haberzettl, J. Haidenbauer, C. Hanhart, S. Krewald, U.-G. Meißner, and K. Nakayama, *in preparation*

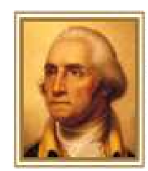
On the importance of maintaining gauge invariance

- **Preliminary results for $\gamma N \to \pi N$:**

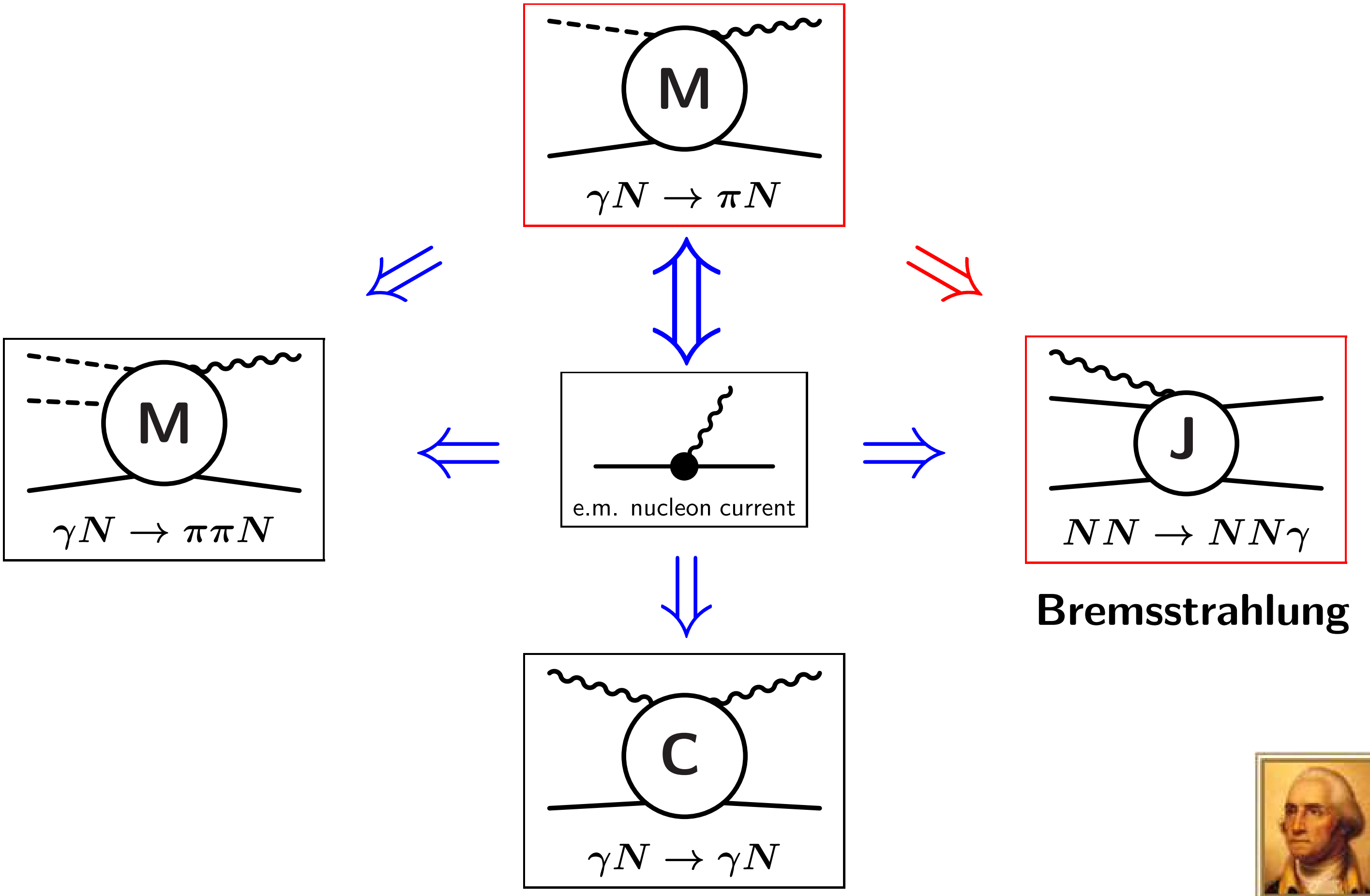


Dashed green curves: w/o M_c^μ

F. Huang, M. Döring, H. Haberzettl, J. Haidenbauer, C. Hanhart, S. Krewald, U.-G. Meißner, K. Nakayama, *to be published* (2011)



Dynamical Links between Photoprocesses — Bremsstrahlung

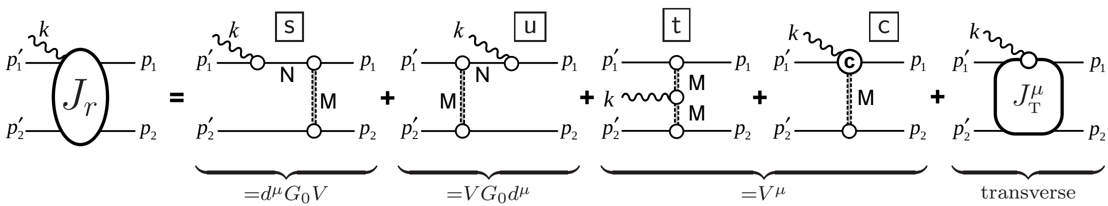


Bremsstrahlung $NN \to NN\gamma$

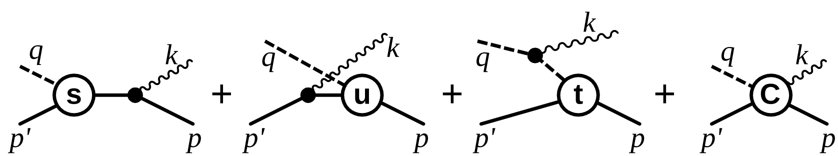
- **Bremsstrahlung Current:**

$$J^\mu_{\mathsf{B}} = (TG_0 + 1)J^\mu_r(1 + G_0 T)$$

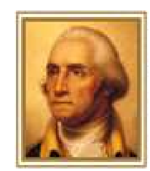
T: NN T-matrix



- **Compare the photon processes along the top nucleon line above to the meson production diagrams below.**



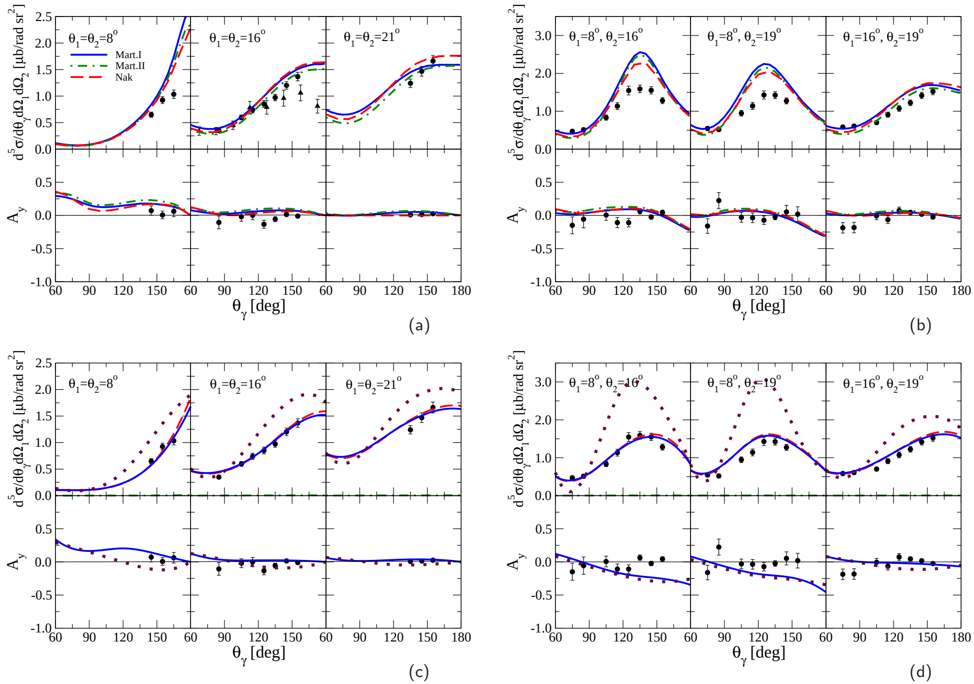
$\Rightarrow$ Essential parts of the process can be described as a meson capture process — i.e., as an inverse photoproduction process — in the presence of a spectator nucleon.



Bremsstrahlung $NN \to NN\gamma$

K. Nakayama, HH, PRC **80**, 051001(R) (2009)

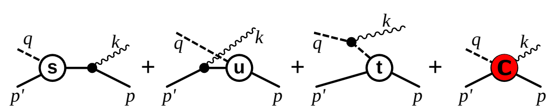
- **Application to KVI data. — Or: Resolving a longstanding problem:**

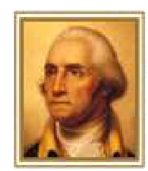


Old

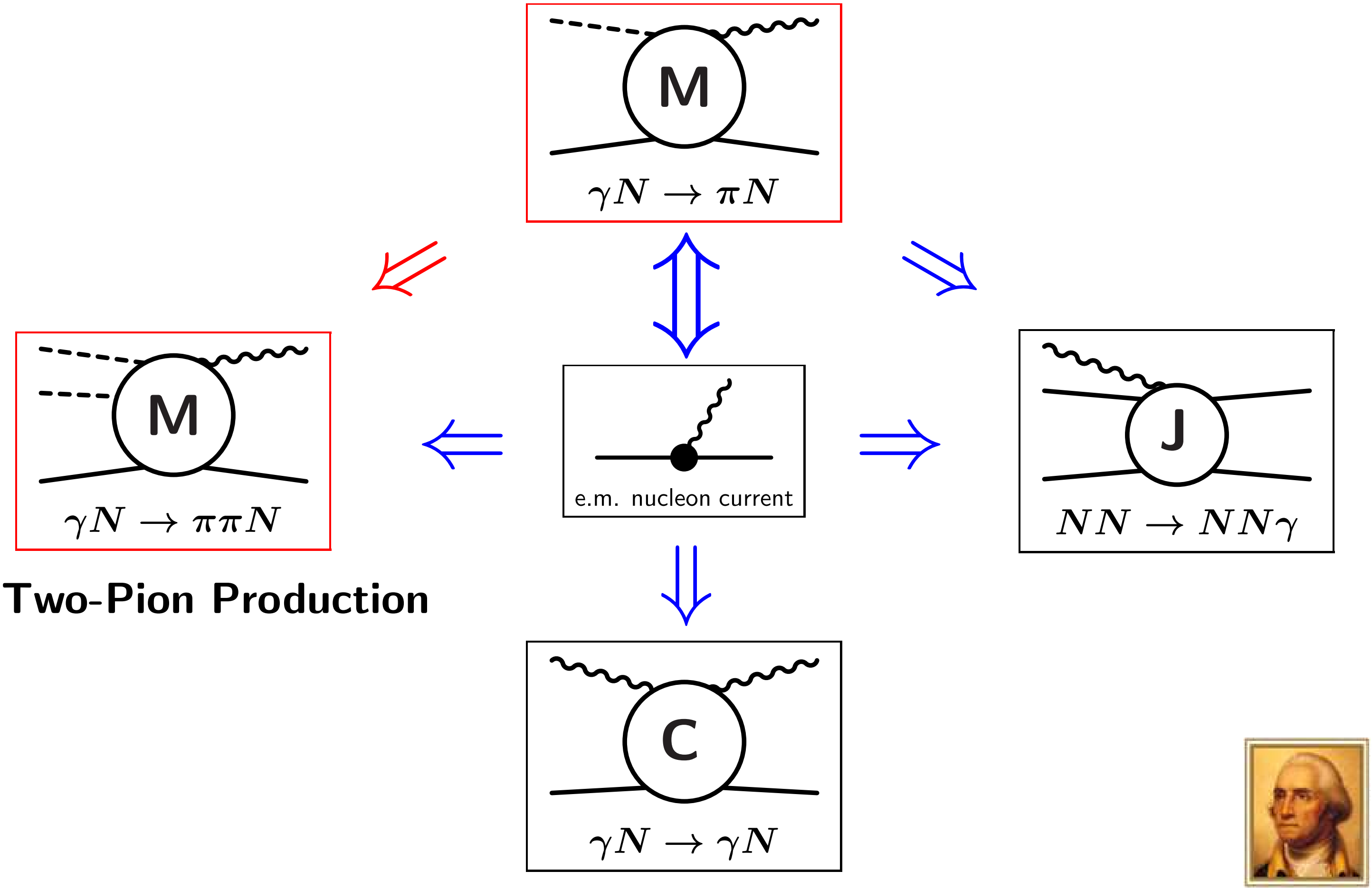
New

- Inclusion of the four-point interaction current from meson photoproduction brings about a dramatic improvement.



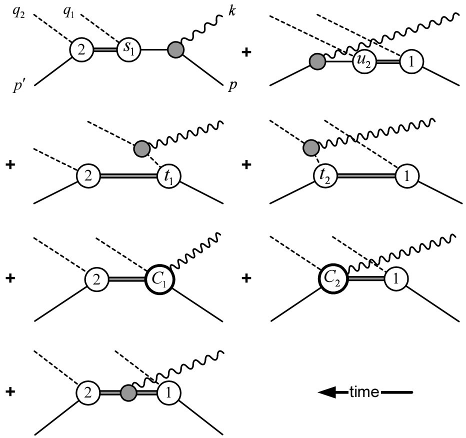


Dynamical Links between Photoprocesses — Two-Pion Production

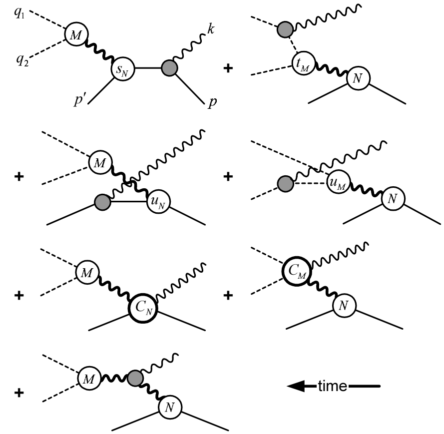


Two-Pion Production

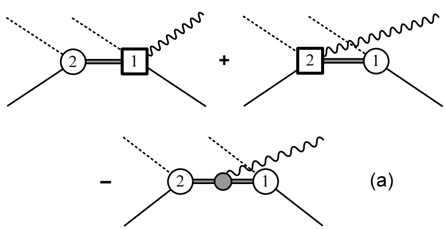
Basic Two-pion Production Mechanisms



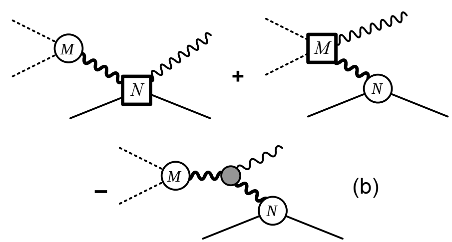
(a)



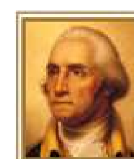
(b)



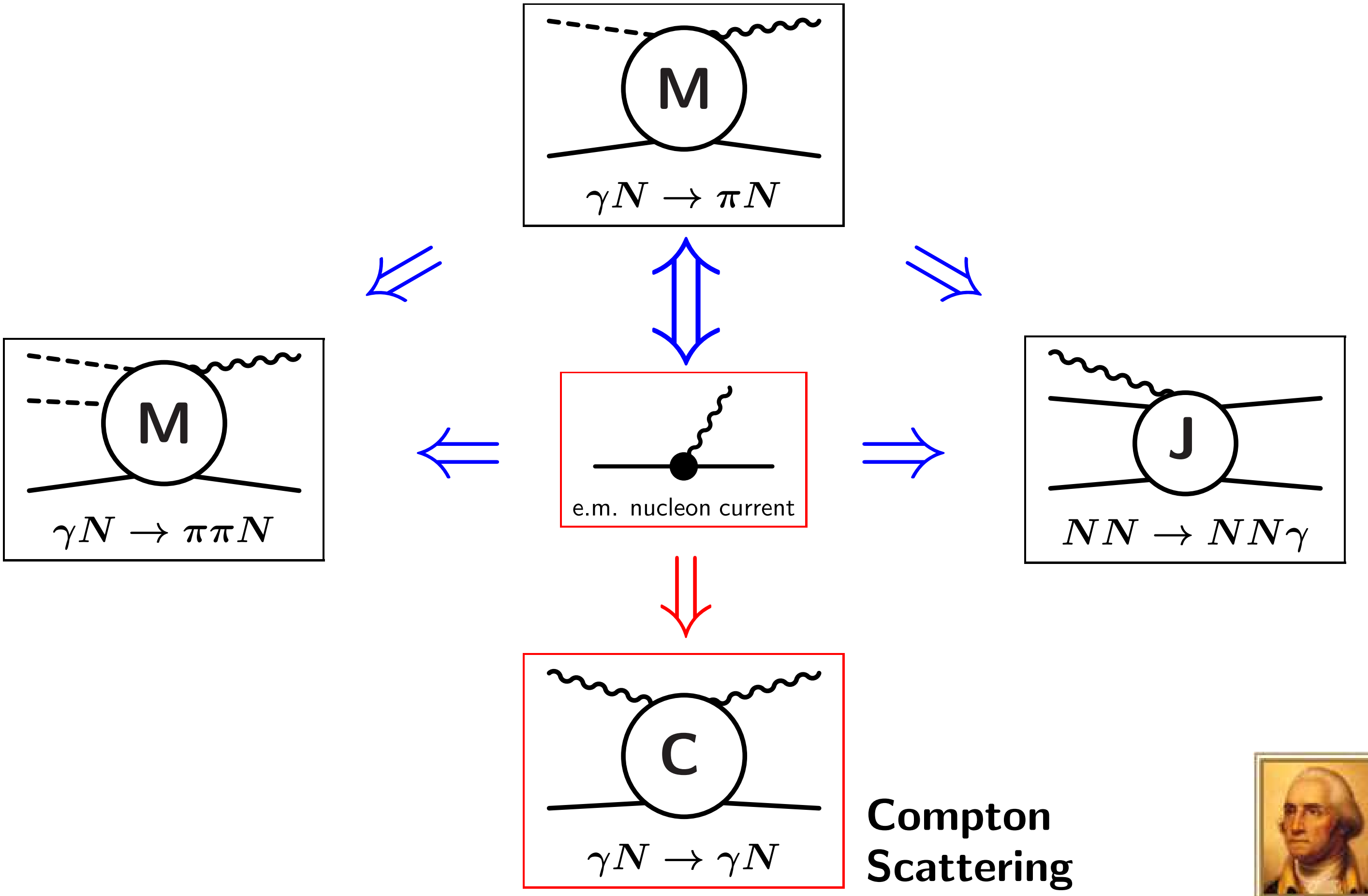
(a)



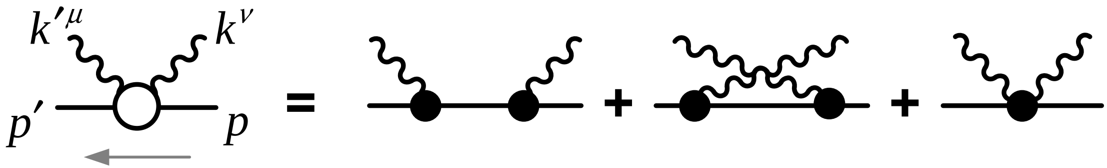
(b)



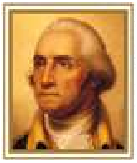
Dynamical Links between Photoprocesses — Compton Scattering



Compton Scattering $\gamma N \to \gamma N$

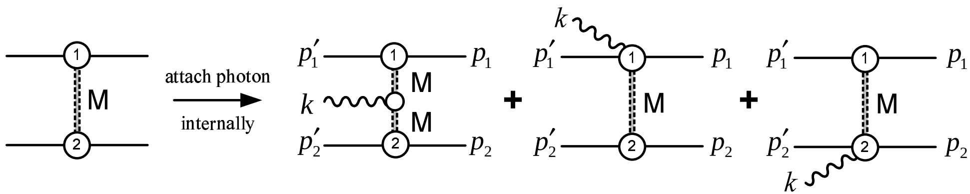


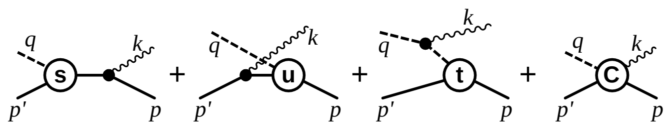
- s- and u-channel terms employ dressed current just described.
- Contact term constrained by gauge invariance.

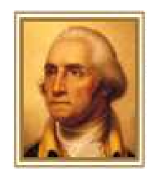


Conclusions

- There exists a very close relationship between the dressed nucleon current and the pion photoproduction current.
- Exploiting this relationship suggests physically meaningful approximations that work, despite the enormous complexity of the exact formalism.
- Maintaining full gauge invariance (as opposed to mere current conservation) is not a luxury but a necessity for the correct microscopic description of the reaction dynamics.
- Requiring gauge invariance in the form of *off-shell* (generalized) Ward-Takahashi identities for each subprocess provides a powerful tool for constraining the contributing mechanisms *and* ensuring overall gauge invariance as a matter of course.
  Case in point:







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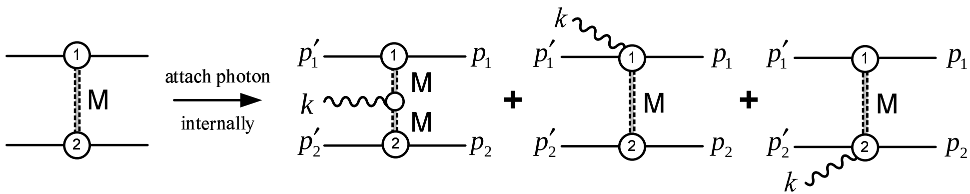

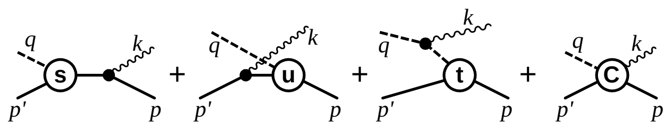

Thank you!